Sec. 5.1 Logarithms and Their Properties

Logarithmic Function: of the form $y = \log_a x$ if and only if $x = a^y$. The domain of the log function is $x > 0$.

Logarithmic functions solves problems such as $10^t = 2500$ without graphing.

We simply use: $t = \log_{10} 2500$

In the previous example it is answering the question: *Ten to what power equals 2500?*

-To solve, you must be able to write expression back and forth between logarithmic form and exponential form.

Ex. Change each expression to either log form or exponential form.

a. $\log_{1/2}|x| = 5$ $\quad (\frac{1}{2})^5 = |x|$

b. $h^4 = 24$ $\quad 4 = \log_h 24$

c. $e^n = 10$ $\quad n = \log_e 10$

d. $\log_e b = -3$ $\quad e^{-3} = b$

e. $12^2 = n$ $\quad 2 = \log_{12} n$

f. $\log_3 81 = x$ $\quad 3^x = 81$

g. $6^x = 216$ $\quad x = \log_6 216$

h. $\log_{121} x = \frac{1}{2}$ $\quad 121^{\frac{1}{2}} = x$

Ex. Find the exact value of the following:

a. $\log_2 16 = c$

$2^c = 16$

$2^c = 2^4$

$\boxed{c = 4}$

b. $\log_3 \frac{1}{27} = c$

$3^c = \frac{1}{27}$

$3^c = \frac{1}{3^3}$

$3^c = 3^{-3}$

$\boxed{c = -3}$

Ex. Without a calculator, evaluate the following, if possible:

(a) $\log 1 = x$

$\log_{10} 1 = x$

$10^x = 1$

$\boxed{x = 0}$

(c) $\log 1{,}000{,}000 = x$

$\log_{10} 1{,}000{,}000 = x$

$10^x = 1{,}000{,}000$

$\boxed{x = 6}$

(d) $\log 0.001 = x$

$\log_{10} .001 = x$

$10^x = .001$

$10^x = \frac{1}{1000}$

$10^x = \frac{1}{10^3}$

$\boxed{x = -3}$

(f) $\log(-100) = x$

$10^x = -100$

NOT POSSIBLE

$\boxed{\text{UNDEFINED}}$

Properties of Common Logs:

$\log(10^x) = x$ for all x

$10^{\log x} = x$ for $x > 0$

$\log 1 = 0$

$\log 10 = 1$

For *a* and *b* both positive and any value of *t*:

$\log(ab) = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log(b^t) = t \cdot \log b$

Ex. Solve $100 \cdot 2^t = 337{,}000{,}000$ for t.

$\frac{100}{100} \quad \frac{100}{100}$

$2^t = 3{,}370{,}000$

$\log 2^t = \log 3{,}370{,}000$

$\frac{t \log 2}{\log 2} = \frac{\log 3{,}370{,}000}{\log 2}$

$t = 21.684$

NOTE: The domain of log function = the range of an exponential function = (0, ∞). The range of a log function = the domain of an exponential function = (-∞, ∞).

Ex. Find the domain of the following log functions:

a. $f(x) = \log_2(x+3)$

$x + 3 > 0$

$x > -3$

b. $g(x) = \log_5\left(\frac{1+x}{1-x}\right)$ ← or graph

$\frac{1+x}{1-x} > 0$ ← graph

$-1 < x < 1$

c. $h(x) = \log_{1/2}|x|$

$|x| > 0$

All reals except

$x \neq 0$

Natural Log Function: y = ln x if and only if $x = e^y$ $(\ln x = \log_e x)$

Common Log Function: f(x) = log x if and only if $x = 10^y$ $(\log x = \log_{10} x)$

$\ln 1 = 0$

$\ln e = 1$

$\ln(e^x) = x$ } Inverses

$e^{\ln x} = x$

Ex. Solve the following equations:

a. $e^{2x} = 5$

$\ln e^{2x} = \ln 5$

$\frac{2x}{2} = \frac{\ln 5}{2}$

$x = .805$

b. $\log_3(4x-7) = 2$

$3^2 = 4x - 7$

$9 = 4x - 7$

$16 = 4x$

$4 = x$

c. $\log_x 64 = 2$

$x^2 = 64$

$x^2 = 8^2$

$x = 8$

Ex. The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk R of having an accident while driving a car can be modeled by the equation $R = 6e^{kx}$ where x is the variable concentration of alcohol in the blood and k is a constant.

a. Suppose that a concentration of alcohol in the blood of .04 results in a 10% risk of an accident. Find the constant k in the equation. Graph $R = 6e^{kx}$ using this value of k.

$\frac{10}{6} = \frac{6e^{K(.04)}}{6}$

$\ln \frac{5}{3} = \ln e^{K(.04)}$

$\frac{\ln(\frac{5}{3})}{.04} = \frac{.04K}{.04}$

$K = 12.77$

$R = 6e^{12.77x}$

b. Using this value of k, what is the risk if the concentration is .17?

$R = 6e^{12.77(.17)}$

$R = 52.60\%$

c. Using the same value of k, what concentration of alcohol corresponds to a risk of 100%?

$100 = 6e^{12.77x}$

$\frac{100}{6} = e^{12.77x}$

$\ln(\frac{50}{3}) = \ln(e^{12.77x})$

$\frac{\ln \frac{50}{3}}{12.77} = \frac{12.77x}{12.77}$

$.22 = x$

d. If the law asserts that anyone with a risk of having an accident of 20% or more should not have driving privileges, at what concentration of alcohol in the blood should a driver be arrested and charged with DUI?

$\frac{20}{6} = \frac{6e^{12.77x}}{6}$

$\ln(\frac{10}{3}) = \ln(e^{12.77x})$

$\ln \frac{10}{3} = 12.77x$

$.09 = x$

HW: pg 185-187, #1-3, 8, 9, 11, 14-17, 20, 23, 30, 32, 33, 35-44, 48 — 39

Evaluate to thousandths